Origami-Constructing a Waterbomb Molecule

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Abstract: The conventional waterbomb base, named for the traditional model that is folded from it, has been used as a starting point for many designs. Its symmetrical body is useful for many models, since it provides four identical flaps as a starting point. However, this can be a constraint, and the need for modification will arise. Occasionally, we may wish to increase or decrease the amount and sizes of flaps, but keep the essential form: a molecule (defined as a crease pattern that can collapse a polygon flat such that all its edges lie on a line) with folds meeting at one point.

Luckily, the waterbomb base is only the simplest case of a family of molecules, and can be altered to any proportion of flap lengths by solving for the height in the derived equation

$$0 = \sum_{m=1}^{\left(\frac{n+n \bmod 2}{2}\right)} \left((-1)^{m+1}(\sigma^n_{2m-1})(h^{n+n \bmod 2-2m})\right),$$

where h is the height of the molecule, and σ^n_k is the elementary symmetric polynomial (the sum of all the possible products of k side lengths out of all n). This equation produces origami-constructible solutions (from quartic polynomials) for up to ten flaps, and indicates the inconstructibility of general higher-order molecules.

Although for most large molecules there will be no practical way to construct the height with absolute accuracy, the solutions for 3, 4, and 5 flaps are useful tools for optimizing models designed using tiles or circle packing. They can be used, while scaling and shuffling, to replace fractured polygons and crease-rich molecules (gusset, arrowhead…) by simple waterbomb molecules. Avoiding the use of cumbersome arrangements to fill in a packing can greatly simplify a design, producing less layers and requiring fewer and simpler folds.